647510 **BSC2MatC201x** **Seat No :_____**

B.Sc. Semester - 2 (*CBCS*) Examination
March/April – 2023 (New Course)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) **(10)**

1. Get the equation of tangent plane and normal to the sphere $x^2 + y^2 + z^2 = a^2$ at the point (α, β, γ).

2. Define: Right circular cylinder. Find the equation of right circular cylinder whose guiding curve is circle $x^2 + y^2 + z^2 = 9; x - y + z = 3$.

3. Derive equation of a cylinder of which generator parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0;\ z = 0$.

Q.1(B) Answer the following question. (Any One) **(04)**

1. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.

2. Find the equation of right circular cylinder whose radius is 2 and axis Is $\frac{x-1}{2} = \frac{y}{3} = z - 3$.

Q.2(A) Answer the following questions. (Any Two) **(10)**

1. If the function $z = f(x, y)$ possesses the first order partial derivatives in the domain D and $x = g(u, v), y = h(u, v)$ also possesses its first order partial derivatives then show that $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$ and $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$.

2. If $u = sin^{-1}(x^2 + y^2)^{\frac{1}{5}}$ then show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2}{5}tanu$ **(ii)** $x^2\frac{\partial^2 u}{\partial x^2} + 2hxy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{25}tanu(2tan^2 u - 3)$.

3. If $f(x, y) = 0$ then derive the formula for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Q.2(B) Answer the following question. (Any One) **(04)**

1. If $u = e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)u$.

2. If $(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$; when $(x, y) \neq (0,0)$

$= 0$; when $(x, y) = (0,0)$

then discuss the continuity of f at (0,0).

Q.3(A) Answer the following questions. (Any Two) **(10)**

1. State and prove Taylor's theorem for the function $z = f(x, y)$.

2. If $\frac{4}{x} + \frac{9}{y} + \frac{16}{z} = 25$ then by Lagrange's method, obtain the value of x, y, z which make $x + y + z$ minimum.

3. Find maxima and minima for the function $f(x, y) = x^2 - 2xy + \frac{y^3}{3} - 3y$.

Q.3(B) Answer the following question. (Any One) **(04)**

1. If there is 1 % error in the measurement of half of major and minor axis of an ellipse then evaluate relative error in measure of its area.
2. If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ then show that $\frac{\partial(u,v,w}{\partial(x,y,z)} = 4$.

Q.4(A) Answer the following questions. (Any Two) **(10)**

1. If $A = [a_{ij}]_{n\times n}, B = [b_{ij}]_{n\times n}$ then prove that:

   (i) $adj(AB) = adjB \cdot adjA$ **(ii)** $(adjA)^T = adjA^T$

2. Define: Hermitian and skew-Hermitian matrix. Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix.
3. Define: Rank of a matrix. Find the rank of a matrix
   $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$.

Q.4(B) Answer the following question. (Any One) **(04)**

1. If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} cosn\alpha & sinn\alpha \\ -sinn\alpha & cosn\alpha \end{bmatrix}; n \in N$.
2. Define: Orthogonal matrix. If $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find the value of l, m, n.

Q.5(A) Answer the following questions. (Any Two) **(10)**

1. Prove that every square matrix satisfies its characteristic equation.
2. Find eigen value and eigen vector of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
3. Find a and b such that the equations
   $x - 2y + 3z = 1, 3x + y - z = 4, 2x - 2y + az = b$
   have **(i)** one solution **(ii)** no solution **(iii)** an infinite number of solutions.

Q.5(B) Answer the following question. (Any One) **(04)**

1. Define: Characteristic equation of a matrix. Find characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.
2. Solve the equations: $x + 3y + 2z = 0, x + 4y + 3z = 0, x + 5y + 4z = 0$.

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